[10-11-16-S12]

EXERCISES 2.2

1–34 ■ Differentiate each function.

1. $f(x) = x^2 - 10x + 100$
2. $g(x) = x^{100} + 50x + 1$
3. $V(r) = \frac{4}{3}\pi r^3$
4. $s(t) = t^8 + 6t^7 - 18t^2 + 2t$
5. $F(x) = (16x)^3$
6. $G(y) = (y^2 + 1)(2y - 7)$
7. $Y(t) = 6t^{-9}$
8. $R(x) = \dfrac{\sqrt{10}}{x^7}$
9. $g(x) = x^2 + \dfrac{1}{x^2}$
10. $f(t) = \sqrt{t} - \dfrac{1}{\sqrt{t}}$
11. $h(x) = \dfrac{x+2}{x-1}$
12. $f(u) = \dfrac{1-u^2}{1+u^2}$
13. $G(s) = (s^2 + s + 1)(s^2 + 2)$
14. $H(t) = \sqrt[3]{t}\,(t + 2)$
15. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$
16. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$
17. $y = \sqrt{5x}$
18. $y = x^{4/3} - x^{2/3}$
19. $y = \dfrac{1}{x^4 + x^2 + 1}$
20. $y = x^2 + x + x^{-1} + x^{-2}$
21. $y = ax^2 + bx + c$
22. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2}$
23. $y = \dfrac{3t - 7}{t^2 + 5t - 4}$
24. $y = \dfrac{4t + 5}{2 - 3t}$
25. $y = x + \sqrt[5]{x^2}$
26. $y = x^4 - \sqrt[4]{x}$
27. $u = x^{\sqrt{2}}$
28. $u = \sqrt[3]{t^2} + 2\sqrt{t^3}$
29. $v = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}}$
30. $v = \dfrac{6}{\sqrt[3]{t^5}}$
31. $f(x) = \dfrac{x}{x + \dfrac{c}{x}}$
32. $f(x) = \dfrac{ax + b}{cx + d}$
33. $f(x) = \dfrac{x^5}{x^3 - 2}$
34. $s = \sqrt{t}\,(t^3 - \sqrt{t} + 1)$

35. The general polynomial of degree n has the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where $a_n \neq 0$. Find the derivative of P.

36–39 ■ Find the equation of the tangent line to the given curve at the specified point.

36. $y = \dfrac{x}{x - 3}$, $(6, 2)$
37. $y = x + \dfrac{4}{x}$, $(2, 4)$
38. $y = x^{5/2}$, $(4, 32)$
39. $y = x + \sqrt{x}$, $(1, 2)$

40. (a) The curve $y = x/(1 + x^2)$ is called a **serpentine**. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.
    (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

41. (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$.
    (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

42. (a) If $f(x) = x/(x^2 - 1)$, find $f'(x)$.
    (b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

43. (a) If $f(x) = 3x^{15} - 5x^3 + 3$, find $f'(x)$.
    (b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

44. Find the equations of the tangent lines to the curve $y = (x - 1)/(x + 1)$ that are parallel to the line $x - 2y = 1$.

45. At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

46. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

47. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

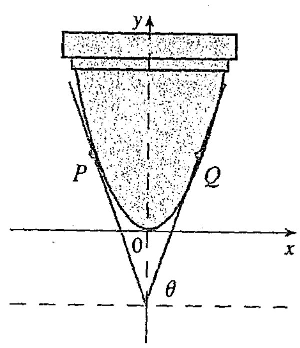
48. Draw a diagram to show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$. Find the coordinates of the points where these tangent lines intersect the parabola.

49. How many tangent lines to the curve $y = x/(x + 1)$ pass through the point $(1, 2)$? At which points do these tangent lines touch the curve?

50. Find the equations of both lines through the point $(2, -3)$ that are tangent to the parabola $y = x^2 + x$.

51. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

52. A manufacturer of cartridges for stereo systems has designed a stylus with parabolic cross-section as shown in the figure. The equation of the parabola is $y = 16x^2$, where



x and y are measured in millimeters. If the stylus sits in a record groove whose sides make an angle of θ with the horizontal direction, where $\tan\theta = 1.75$, find the points of contact P and Q of the stylus with the groove.

53–56 ■ Find the equation of the normal line to the curve at the given point. (The **normal line** to a curve C at a point P is, by definition, the line that passes through P and is perpendicular to the tangent line to C at P.) Also sketch the curve and the normal line in Exercises 53–55.

53. $y = 1 - x^2$, $(2, -3)$ **54.** $y = \dfrac{1}{x-1}$, $(2, 1)$

55. $y = \sqrt[3]{x}$, $(-8, -2)$

56. $y = f(x)$, $(a, f(a))$

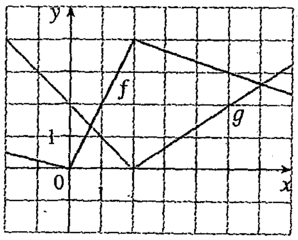
57. At what point on the curve $y = x^4$ does the normal line have slope 16?

58. Where does the normal line to the parabola $y = x - x^2$ at the point $(1, 0)$ intersect the parabola a second time? Illustrate with a sketch.

59. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the values of (a) $(fg)'(5)$, (b) $(f/g)'(5)$, and (c) $(g/f)'(5)$.

60. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$, find the following numbers:
(a) $(f + g)'(3)$ (b) $(fg)'(3)$
(c) $(f/g)'(3)$ (d) $\left(\dfrac{f}{f-g}\right)'(3)$

61. If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.
(a) Find $u'(1)$. (b) Find $v'(5)$.



62. If f is a differentiable function, find an expression for the derivative of each of the following functions.
(a) $y = x^2 f(x)$ (b) $y = \dfrac{f(x)}{x^2}$
(c) $y = \dfrac{x^2}{f(x)}$ (d) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

63. (a) Use the Product Rule twice to prove that if f, g, and h are differentiable, then

$$(fgh)' = f'gh + fg'h + fgh'$$

(b) Taking $f = g = h$ in part (a), show that

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x)$$

64–66 ■ Use Exercise 63 to differentiate each function.

64. $y = (x + 5)(x^2 + 7)(x - 3)$

65. $y = \sqrt{x}(x^4 + x + 1)(2x - 3)$

66. $y = (x^4 + 3x^3 + 17x + 82)^3$

67. Let

$$f(x) = \begin{cases} 2 - x & \text{if } x \le 1 \\ x^2 - 2x + 2 & \text{if } x > 1 \end{cases}$$

Is f differentiable at 1? Sketch the graphs of f and f'.

68. At what numbers is the following function g differentiable?

$$g(x) = \begin{cases} -1 - 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 1 \\ x & \text{if } x > 1 \end{cases}$$

Give a formula for g' and sketch the graphs of g and g'.

69. (a) For what values of x is the function $f(x) = |x^2 - 9|$ differentiable? Find a formula for f'.
(b) Sketch the graphs of f and f'.

70. Where is the function $h(x) = |x - 1| + |x + 2|$ differentiable? Give a formula for h' and sketch the graphs of h and h'.

71. For what values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = 2$?

72. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find the values of m and b that make f differentiable everywhere.

73. An easy proof of the Quotient Rule can be given if we make the prior assumption that $F'(x)$ exists, where $F = f/g$. Write $f = Fg$; then differentiate using the Product Rule and solve the resulting equation for F'.

74. A tangent line is drawn to the hyperbola $xy = c$ at a point P.
(a) Show that the midpoint of the line segment cut off this tangent line by the coordinate axes is P.
(b) Show that the triangle formed by the tangent line and the coordinate axes always has the same area, no matter where P is located on the hyperbola.

75. Evaluate $\displaystyle\lim_{x\to 1} \frac{x^{1000} - 1}{x - 1}$.

76. Draw a diagram showing two perpendicular lines that intersect on the y-axis and are both tangent to the parabola $y = x^2$. Where do these lines intersect?

This is an illustration of the fact that part of the power of mathematics lies in its abstractness. A single abstract mathematical concept (such as the derivative) can have different interpretations in each of the sciences. When we develop the properties of the mathematical concept once and for all, we can then turn around and apply these results to all of the sciences. This is much more efficient than developing properties of special concepts in each separate science. The French mathematician Joseph Fourier (1768–1830) put it succinctly: "Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them."

EXERCISES 2.3

1–6 ■ A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.
(b) What is the velocity after 2 s?
(c) When is the particle at rest?
(d) When is the particle moving in the positive direction?
(e) Find the total distance traveled during the first 4 s.
(f) Draw a diagram like Figure 2 to illustrate the motion of the particle.

1. $f(t) = t^2 - 6t + 9$

2. $f(t) = 4t^3 - 9t^2 + 6t + 2$

3. $f(t) = 2t^3 - 9t^2 + 12t + 1$

4. $f(t) = t^4 - 4t + 1$

5. $s = \dfrac{t}{t^2 + 1}$ **6.** $s = \sqrt{t}\,(5 - 5t + 2t^2)$

7. The position function of a particle is given by $s = t^3 - 4.5t^2 - 7t$, $t \geq 0$. When does the particle reach a velocity of 5 m/s?

8. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$.
(a) What is the maximum height reached by the ball?
(b) What is the velocity of the ball when it is 96 ft above the ground on its way up? on its way down?

9. (a) Find the average rate of change of the volume of a cube with respect to its edge length x as x changes from
(i) 5 to 6 (ii) 5 to 5.1 (iii) 5 to 5.01
(b) Find the instantaneous rate of change when $x = 5$.
(c) Show that the rate of change of the volume of a cube with respect to its edge length (at any x) is equal to half the surface area of the cube.

10. (a) Find the average rate of change of the area of a circle with respect to its radius r as r changes from
(i) 2 to 3 (ii) 2 to 2.5 (iii) 2 to 2.1
(b) Find the instantaneous rate of change when $r = 2$.
(c) Show that the rate of change of the area of a circle with respect to its radius (at any r) is equal to the circumference of the circle.

11. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s. Find the rate at which the area within the circle is increasing after (a) 1 s, (b) 3 s, and (c) 5 s.

12. (a) The volume of a growing spherical cell is $V = \frac{4}{3}\pi r^3$, where the radius r is measured in micrometers ($1\ \mu m = 10^{-6}$ m). Find the average rate of change of V with respect to r when r changes from
(i) 5 to 8 μm (ii) 5 to 6 μm (iii) 5 to 5.1 μm
(b) Find the instantaneous rate of change of V with respect to r when $r = 5\ \mu$m.

13. A spherical balloon is being inflated. Find the rate of increase of the surface area ($S = 4\pi r^2$) with respect to the radius r when r is (a) 1 ft, (b) 2 ft, and (c) 3 ft.

14. Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area.

15. The mass of the part of a metal rod that lies between its left end and a point x meters to the right is $3x^2$ kg. Find the linear density (see Example 2) when x is (a) 1 m, (b) 2 m, and (c) 3 m.

16. If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 min, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 5000\left(1 - \frac{t}{40}\right)^2 \qquad 0 \leq t \leq 40$$

Find the rate at which water is draining from the tank after (a) 5 min, (b) 10 min, and (c) 20 min.

17. The quantity of charge Q in coulombs (C) that has passed through a surface up to time t (measured in seconds) is given by $Q(t) = t^3 - 2t^2 + 6t + 2$. Find the current when (a) $t = 0.5$ s and (b) $t = 1$ s. [See Example 3. The unit of current is an ampere (1 A = 1 C/s).]

18. Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies. If the bodies are moving, find the rate of change of F with respect to r.

EXAMPLE 4 Differentiate $f(x) = \dfrac{\sec x}{1 + \tan x}$.

SOLUTION The Quotient Rule gives

$$\begin{aligned} f'(x) &= \frac{(1 + \tan x)\, D \sec x - \sec x\, D(1 + \tan x)}{(1 + \tan x)^2} \\ &= \frac{(1 + \tan x) \sec x \tan x - \sec x \cdot \sec^2 x}{(1 + \tan x)^2} \\ &= \frac{\sec x [\tan x + \tan^2 x - \sec^2 x]}{(1 + \tan x)^2} \\ &= \frac{\sec x (\tan x - 1)}{(1 + \tan x)^2} \end{aligned}$$

In simplifying the answer we have used the identity $\tan^2 x + 1 = \sec^2 x$.

EXERCISES 2.4

1–16 ■ Find each limit.

1. $\lim_{x \to 0} (x^2 + \cos x)$
2. $\lim_{x \to 0} \cos(\sin x)$
3. $\lim_{x \to \pi/3} (\sin x - \cos x)$
4. $\lim_{x \to \pi} x^2 \sec x$
5. $\lim_{x \to \pi/4} \dfrac{\sin x}{3x}$
6. $\lim_{x \to 0} \dfrac{\sin x}{3x}$
7. $\lim_{t \to 0} \dfrac{\sin 5t}{t}$
8. $\lim_{t \to 0} \dfrac{\sin 8t}{\sin 9t}$
9. $\lim_{\theta \to 0} \dfrac{\sin(\cos \theta)}{\sec \theta}$
10. $\lim_{\theta \to 0} \dfrac{\cos \theta - 1}{\sin \theta}$
11. $\lim_{x \to \pi/4} \dfrac{\tan x}{4x}$
12. $\lim_{x \to 0} \dfrac{\tan x}{4x}$
13. $\lim_{\theta \to 0} \dfrac{\sin^2 \theta}{\theta}$
14. $\lim_{h \to 0} \dfrac{\sin 5h}{\tan 3h}$
15. $\lim_{x \to 0} \dfrac{\tan 3x}{3 \tan 2x}$
16. $\lim_{t \to 0} \dfrac{\sin^2 3t}{t^2}$
17. Prove that $\dfrac{d}{dx} (\csc x) = -\csc x \cot x$.
18. Prove that $\dfrac{d}{dx} (\sec x) = \sec x \tan x$.
19. Prove that $\dfrac{d}{dx} (\cot x) = -\csc^2 x$.

20–31 ■ Find $\dfrac{dy}{dx}$.

20. $y = \cos x - 2 \tan x$
21. $y = \sin x + \cos x$
22. $y = x \csc x$
23. $y = \csc x \cot x$
24. $y = \dfrac{\sin x}{1 + \cos x}$
25. $y = \dfrac{\tan x}{x}$
26. $y = \dfrac{\tan x - 1}{\sec x}$
27. $y = \dfrac{x}{\sin x + \cos x}$
28. $y = 2x(\sqrt{x} - \cot x)$
29. $y = x^{-3} \sin x \tan x$
30. $y = x \sin x \cos x$
31. $y = \dfrac{x^2 \tan x}{\sec x}$

32–34 ■ Find the equation of the tangent line to the given curve at the specified point.

32. $y = 2 \sin x, \quad (\pi/6, 1)$
33. $y = \tan x, \quad (\pi/4, 1)$
34. $y = \sec x - 2 \cos x, \quad (\pi/3, 1)$
35. (a) Find an equation of the tangent line to the curve $y = x \cos x$ at the point $(\pi, -\pi)$.
    (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.
36. (a) If $f(x) = 2x + \cot x$, find $f'(x)$.
    (b) Check to see that your answer to part (a) is reasonable by graphing both f and f' for $0 < x < \pi$.
37. For what values of x does the graph of $f(x) = x + 2 \sin x$ have a horizontal tangent?
38. Find the points on the curve $y = (\cos x)/(2 + \sin x)$ at which the tangent is horizontal.
39. A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let

x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$?

40. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a constant called the *coefficient of friction*.
(a) Find the rate of change of F with respect to θ.
(b) When is this rate of change equal to 0?
(c) If $W = 50$ lb and $\mu = 0.6$, draw the graph of F as a function of θ and use it to locate the value of θ for which $dF/d\theta = 0$. Is the value consistent with your answer to part (b)?

41. If we put $\theta = 2x$ in the identity $\cos 2x = 1 - 2\sin^2 x$, it becomes $\cos\theta = 1 - 2\sin^2(\theta/2)$. Use this identity to give an alternative proof of Corollary 7.

42. Prove, using the definition of derivative, that if $f(x) = \cos x$, then $f'(x) = -\sin x$.

43–54 ▪ Find each limit.

43. $\displaystyle\lim_{x\to 0} \frac{\cot 2x}{\csc x}$

44. $\displaystyle\lim_{x\to 0} \frac{1 - \cos x}{2x^2}$

45. $\displaystyle\lim_{x\to \pi} \frac{\tan x}{\sin 2x}$

46. $\displaystyle\lim_{x\to \pi/4} \frac{\sin x - \cos x}{\cos 2x}$

47. $\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\theta + \tan\theta}$

48. $\displaystyle\lim_{x\to 1} \frac{\sin(x - 1)}{x^2 + x - 2}$

49. $\displaystyle\lim_{x\to 0} \frac{\cos x \sin x - \tan x}{x^2 \sin x}$

50. $\displaystyle\lim_{y\to 0} \left(\lim_{x\to 0} \frac{\cos x \sin y}{x - y}\right)$

51. $\displaystyle\lim_{x\to 0} \frac{\sin(\sin x)}{\sin x}$

52. $\displaystyle\lim_{x\to 0} \frac{\sin(\sin x)}{x}$

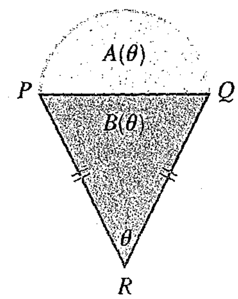
53. $\displaystyle\lim_{x\to \infty} \cos\frac{1}{x}$

54. $\displaystyle\lim_{x\to \infty} \left(x - x\cos\frac{1}{x}\right)$

55. Differentiate each trigonometric identity to obtain a new (or familiar) identity.
(a) $\tan x = \dfrac{\sin x}{\cos x}$
(b) $\sec x = \dfrac{1}{\cos x}$
(c) $\sin x + \cos x = \dfrac{1 + \cot x}{\csc x}$

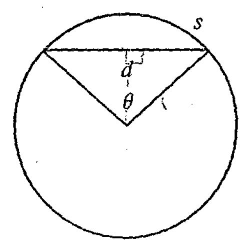
56. A semicircle with diameter PQ sits on an isosceles triangle PQR to form a region shaped like an ice cream cone, as shown in the figure. If $A(\theta)$ is the area of the semicircle and $B(\theta)$ is the area of the triangle, find

$$\lim_{\theta\to 0^+} \frac{A(\theta)}{B(\theta)}$$



57. The figure shows a circular arc of length s and a chord of length d, both subtended by a central angle θ. Find

$$\lim_{\theta\to 0^+} \frac{s}{d}$$

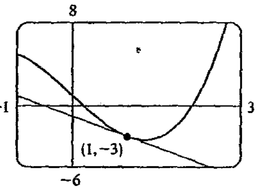


CHAPTER 2

Exercises 2.1 ■ page 121

1. $7, 7x - y - 12 = 0$

3. (a) $-2, 2x + y + 1 = 0$
(b)



5. -2 m/s
7. $1 - 4a$
9. $-1/(2a - 1)^2$
11. $1/(3 - a)^{3/2}$
13. $f(x) = \sqrt{x}, a = 1$
15. $f(x) = x^9, a = 1$

17. $f(x) = \sin x, a = \pi/2$ **19.** $f'(x) = 5, \mathbb{R}, \mathbb{R}$

21. $f'(x) = 3x^2 - 2x + 2, \mathbb{R}, \mathbb{R}$

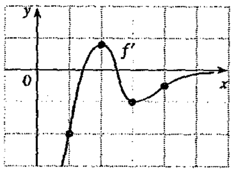
23. $g'(x) = 1/\sqrt{1 + 2x}, [-\frac{1}{2}, \infty), (-\frac{1}{2}, \infty)$

25. $G'(x) = -10/(2 + x)^2, \{x \mid x \neq -2\}, \{x \mid x \neq -2\}$

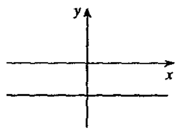
27. $f'(x) = 4x^3, \mathbb{R}, \mathbb{R}$ **29.** $1, 2x, 3x^2, nx^{n-1}, 5x^4$

31. (a) $f'(x) = 1 + 2/x^2$

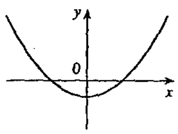
33. (a) -2
(b) 0.8
(c) -1
(d) -0.5



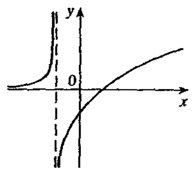
35.



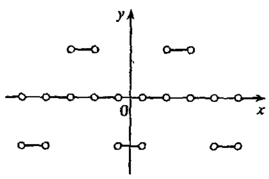
37.



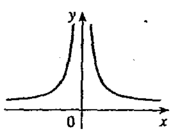
39.



41.



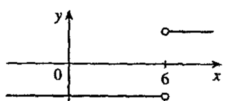
43.



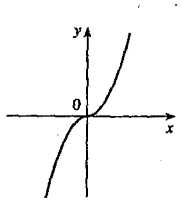
45. 3.296
47. $-5, 4, 8, 9, 5, -0.5, -8$
49. (a) $1/(3a^{2/3})$

51. $-1, 11$ (vertical tangents), 4 (discontinuity), 8 (corner)

53. $f'(x) = \begin{cases} -1 & \text{if } x < 6 \\ 1 & \text{if } x > 6 \end{cases}$
or $f'(x) = \dfrac{x - 6}{|x - 6|}$



55. (a)



(b) All x
(c) $f'(x) = 2|x|$

57. (a) $-5, 5$ **59.** Does not exist **63.** 63°

Exercises 2.2 ■ page 132

1. $f'(x) = 2x - 10$ **3.** $V'(r) = 4\pi r^2$ **5.** $F'(x) = 12288x^2$

7. $Y'(t) = -54t^{-10}$ **9.** $g'(x) = 2x - (2/x^3)$

11. $h'(x) = -3/(x - 1)^2$

13. $G'(s) = (2s + 1)(s^2 + 2) + (s^2 + s + 1)(2s)$
$[= 4s^3 + 3s^2 + 6s + 2]$

15. $y' = \frac{3}{2}\sqrt{x} + (2/\sqrt{x}) - 3/(2x\sqrt{x})$ **17.** $y' = \sqrt{5}/(2\sqrt{x})$

19. $y' = -(4x^3 + 2x)/(x^4 + x^2 + 1)^2$ **21.** $y' = 2ax + b$

23. $y' = (-3t^2 + 14t + 23)/(t^2 + 5t - 4)^2$

25. $y' = 1 + 2/(5\sqrt[5]{x^3})$ **27.** $u' = \sqrt{2}\, x^{\sqrt{2}-1}$

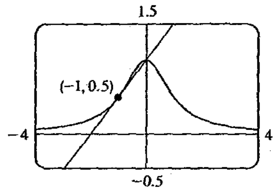
29. $v' = \frac{3}{2}\sqrt{x} - 5/(2x^3\sqrt{x})$ **31.** $f'(x) = 2cx/(x^2 + c)^2$

33. $f'(x) = 2x^4(x^3 - 5)/(x^3 - 2)^2$

35. $P'(x) = na_nx^{n-1} + (n - 1)a_{n-1}x^{n-2} + \cdots + 2a_2x + a_1$

37. $y = 4$ **39.** $3x - 2y + 1 = 0$

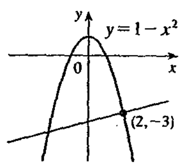
41. (a) $x - 2y + 2 = 0$ (b)

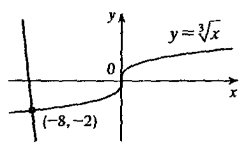


43. (a) $45x^{14} - 15x^2$ **45.** $(4, 8)$

47. $(1, 0), (-\frac{1}{3}, \frac{32}{27})$ **49.** $2, (-2 \pm \sqrt{3}, (1 \mp \sqrt{3})/2)$

53. $x - 4y - 14 = 0$ **55.** $12x + y + 98 = 0$



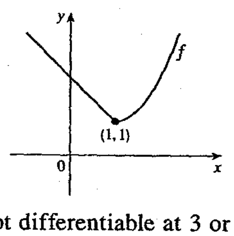


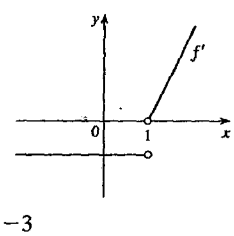
57. $(-\frac{1}{4}, \frac{1}{256})$ **59.** (a) -16 (b) $-\frac{20}{9}$ (c) 20

61. (a) 0 (b) $-\frac{2}{3}$

65. $y' = (x^4 + x + 1)(2x - 3)/(2\sqrt{x})$
$+ \sqrt{x}\,[(4x^3 + 1)(2x - 3) + 2(x^4 + x + 1)]$

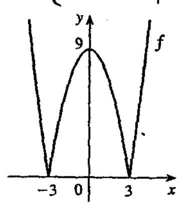
67. No

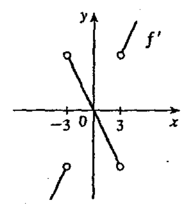




69. (a) Not differentiable at 3 or -3
$f'(x) = \begin{cases} 2x & \text{if } |x| > 3 \\ -2x & \text{if } |x| < 3 \end{cases}$

(b)

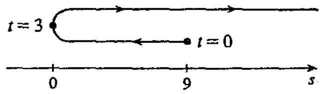




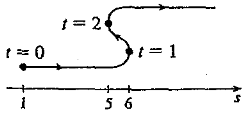
71. $a = -\frac{1}{2}, b = 2$ **75.** 1000

:ises 2.3 ■ page 142

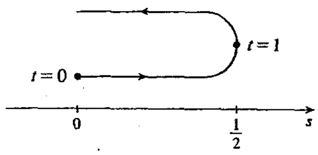
) $2t - 6$ (b) -2 ft/s (c) $t = 3$ (d) $t > 3$
) ft (f)



) $6t^2 - 18t + 12$ (b) 0 ft/s (c) $t = 1, 2$
$\leqslant t < 1,\ t > 2$ (e) 34 ft



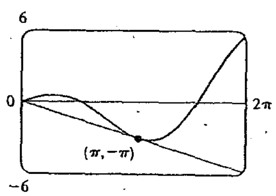
) $(1 - t^2)/(t^2 + 1)^2$ (b) $-\frac{3}{25}$ ft/s (c) $t = 1$
$\leqslant t < 1$ (e) $\frac{13}{17}$ ft (f)



= 4 s
) (i) 91 (ii) 76.51 (iii) 75.1501 (b) 75
a) 7200π cm^2/s (b) $21{,}600\pi$ cm^2/s (c) $36{,}000\pi$ cm^2/s
a) 8π ft^2/ft (b) 16π ft^2/ft (c) 24π ft^2/ft
a) 6 kg/m (b) 12 kg/m (c) 18 kg/m
a) 4.75 A (b) 5 A **19.** (a) $dV/dP = -C/P^2$
a) $a^2k/(akt + 1)^2$ **23.** -0.04
= -92.6 (cm/s)/cm
$'(x) = 1.5 + 0.004x$; \$1.90, \$1.902
$'(x) = 3 + 0.02x + 0.0006x^2$; \$11, \$11.07

:ises 2.4 ■ page 149

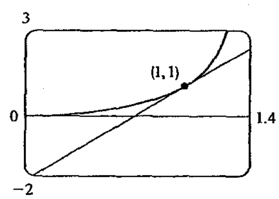
3. $(\sqrt{3} - 1)/2$ **5.** $2\sqrt{2}/(3\pi)$ **7.** 5 **9.** $\sin 1$
$/\pi$ **13.** 0 **15.** $\frac{1}{2}$ **21.** $\cos x - \sin x$
$-\csc x \cot^2 x - \csc^3 x$ **25.** $(x\sec^2 x - \tan x)/x^2$
$\sin x + \cos x + x\sin x - x\cos x)/(1 + \sin 2x)$
$^{-4}\sin x\,(-3\tan x + x + x\sec^2 x)$
$2x\tan x + x^2)/\sec x$ **33.** $4x - 2y = \pi - 2$
a) $y = -x$ (b)



$2n + 1)\pi \pm \pi/3$, n an integer **39.** 5 ft/rad
45. $\frac{1}{2}$ **47.** $\frac{1}{2}$ **49.** -1 **51.** 1 **53.** 1
a) $\sec^2 x = 1/\cos^2 x$
$\sec x \tan x = (\sin x)/\cos^2 x$
$\cos x - \sin x = (\cot x - 1)/\csc x$

Exercises 2.5 ■ page 156

1. $4u(x + 1)$, 48 **3.** $3u^2(1 - 1/x^2)$, 0
5. $F'(x) = 10(x^2 + 4x + 6)^4(x + 2)$
7. $G'(x) = 6(3x - 2)^9(5x^2 - x + 1)^{11}(85x^2 - 51x + 9)$
9. $f'(t) = -16(2t^2 - 6t + 1)^{-9}(2t - 3)$
11. $g'(x) = (2x - 7)/(2\sqrt{x^2 - 7x})$
13. $h'(t) = \frac{3}{2}(t - 1/t)^{1/2}(1 + 1/t^2)$
15. $F'(y) = 39(y - 6)^2/(y + 7)^4$
17. $f'(z) = -\frac{2}{5}(2z - 1)^{-6/5}$
19. $y' = 8(2x - 5)^3(8x^2 - 5)^{-4}(-4x^2 + 30x - 5)$
21. $y' = 3\sec^2 3x$ **23.** $y' = -3x^2\sin(x^3)$
25. $y' = -12\cos x \sin x\,(1 + \cos^2 x)^5$
27. $y' = -\sin(\tan x)\sec^2 x$ **29.** $y' = 0$
31. $y' = -(1/3)\csc(x/3)\cot(x/3)$
33. $y' = 3\sin x\cos x\,(\sin x - \cos x)$
35. $y' = -\cos(1/x)/x^2$ **37.** $y' = 4(\cos 2x)/(1 - \sin 2x)^2$
39. $y' = 6x^2\tan(x^3)\sec^2(x^3)$ **41.** $y' = 0$
43. $y' = [1 + 1/(2\sqrt{x})]/(2\sqrt{x + \sqrt{x}})$
45. $f'(x) = 9[x^3 + (2x - 1)^3]^2(9x^2 - 8x + 2)$
47. $y' = \cos(\tan\sqrt{\sin x})(\sec^2\sqrt{\sin x})[1/(2\sqrt{\sin x})](\cos x)$
49. $y = 0$ **51.** $3x + 16y = 44$
53. (a) $y = \pi x - \pi + 1$ (b)



55. (a) $-1/(x^2\sqrt{1 - x^2})$
57. $((\pi/2) + 2n\pi, 3)$, $((3\pi/2) + 2n\pi, -1)$, n any integer
59. 28 **61.** $v(t) = (5\pi/2)\cos(10\pi t)$ cm/s
63. (a) $dB/dt = (7\pi/54)\cos(2\pi t/5.4)$ (b) 0.16
65. (a) On $(0, \infty)$ (b) $G'(x) = h'(\sqrt{x})/(2\sqrt{x})$
67. (a) $F'(x) = -\sin x\, f'(\cos x)$ (b) $G'(x) = -\sin(f(x))f'(x)$
69. 0 **75.** $f'(x) = x/|x|$
77. $h'(x) = |2x - 1| + 2x(2x - 1)/|2x - 1|$

Exercises 2.6 ■ page 162

1. (a) $y' = -(2x + y + 3)/x$
(b) $y = (5/x) - x - 3$, $y' = -(5/x^2) - 1$
3. (a) $y' = (2x - y)/(x + 4y)$
(b) $y = (-x \pm \sqrt{9x^2 + 24})/4$, $y' = (-1 \pm 9x/\sqrt{9x^2 + 24})/4$
5. $y' = (y - 2x)/(3y^2 - x)$
7. $y' = (18x - x^{-2/3}y^{1/3})/(12y + x^{1/3}y^{-2/3})$
9. $y' = -x^3/y^3$
11. $y' = (y/x) + 2(x - y)^2$ [or $(3x^2 + 1 - 2xy)/(x^2 + 2)$]
13. $y' = [\sin(x - y) + y\cos x]/[\sin(x - y) - \sin x]$
15. $y' = -y/x$
17. $dx/dy = (1 - 4y^3 - 2x^2y - x^4)/(2xy^2 + 4yx^3)$ **19.** $-\frac{1}{6}$
21. $5x + 4y + 16 = 0$ **23.** $y = x$
25. $9x + 13y - 40 = 0$